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Dual doubled geometry

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ABSTRACT

We probe doubled geometry with dual fundamental branes, i.e. solitons. Restricting ourselves first to solitonic branes with more than two transverse directions we find that the doubled geometry requires an effective wrapping rule for the solitonic branes which is dual to the wrapping rule for fundamental branes. This dual wrapping rule can be understood by the presence of Kaluza–Klein monopoles. Extending our analysis to supersymmetric solitonic branes with less than or equal to two transverse directions we show that such solitons are precisely obtained by applying the same dual wrapping rule to these cases as well. This extended wrapping rule cannot be explained by the standard Kaluza–Klein monopole alone. Instead, it suggests the existence of a class of generalized Kaluza–Klein monopoles in ten-dimensional string theory.

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1. Introduction

Over the course of years supergravity has provided a number of key insights into string theory. Examples of such discoveries are the Green–Schwarz anomaly cancellation [1] and the presence of branes in string theory such as the eleven-dimensional supermembrane [2]. A key signature for a p -brane, i.e. a brane with p spatial directions, in string theory is the presence of a $(p+1)$ -form potential in the corresponding supergravity theory. Usually, branes have more than two transverse directions and such branes couple to potentials that describe physical degrees of freedom. These are the so-called “standard” branes and they are well understood. Whenever a brane is standard the dual brane, i.e. the brane that couples to the dual potential, is also standard. The remaining “non-standard” branes are the branes with two transverse directions (“defect”-branes), one transverse direction (“domain-walls”) and no transverse direction at all (“space-filling branes”).¹ The special thing about the defect-branes is that they couple to the duals of scalars parametrizing a coset manifold G/H . This leads to $(D-2)$ -form potentials transforming in the adjoint of G whose curvatures satisfy $\dim H$ non-linear constraints that involve the scalars themselves. Due to this there is no one-to-one correspondence between

a potential and a corresponding defect-brane. Domain-walls and space-filling branes are special in the sense that they couple to potentials that do not describe any physical degrees of freedom. In the case of domain walls the corresponding $(D-1)$ -form potential is dual to an integration constant while the space-filling brane couples to a D -form potential that is not dual to anything at all.

In recent years it has been realized that maximal supergravities can be extended with potentials of rank $D-1$ and D occurring in specific U-duality representations [3–5]. In recent work [6,7] we have developed a criterion to see which of the high-form potentials of rank $D-2$, $D-1$ and D couple to supersymmetric defect-branes, domain walls or space-filling branes, respectively. Our requirement was that a gauge-invariant Wess–Zumino (WZ) term should exist that only involves worldvolume fields that fit into a supermultiplet. This is the minimum requirement for the construction of a kappa-symmetric worldvolume action. The result of our analysis was that not all potentials couple to supersymmetric branes. Furthermore, we found that the supersymmetric non-standard branes do not fill complete U-duality representations. A prime example of this phenomenon are the 8-forms of IIB supergravity. They transform as the **3** of $SL(2, \mathbb{R})$ S-duality but there is only a two-dimensional space of supersymmetric configurations, spanned by the D7-brane and its S-dual [8].²

In this Letter we consider a specific class of branes suggested by supergravity, i.e. the supersymmetric solitons. These are branes whose tension scales with the inverse squared of the string

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¹ It is well known that these non-standard branes are not well-defined when considered as single branes. We will not discuss these issues here and instead only consider whether or not the single brane case is consistent with basic requirements such as gauge invariance and supersymmetry.

² Note that there is a single constraint on the 9-form curvatures. This constraint is needed for the correct counting of physical degrees of freedom in IIB supergravity but not for the correct counting of supersymmetric branes.

coupling constant in the string frame. They are the duals of the fundamental branes whose tension is independent of the string coupling constant. It is well known that a T-duality covariant formulation of the fundamental branes of toroidally compactified string theory with 32 supercharges requires a doubled geometry [9]. In this Letter we will probe this doubled geometry with the dual solitons. Before discussing doubled geometry and solitons we first review in the next section the relation between doubled geometry and fundamental branes and D-branes. For fundamental branes, doubled geometry requires an effective wrapping rule that gives rise to all the fundamental branes in a given dimension starting from ten dimensions. We will then discuss solitonic branes. We will first consider the standard solitons, i.e. those with more than two transverse directions. Probing the doubled geometry with such solitons requires an effective wrapping rule for the solitonic branes which is dual to the wrapping rule for fundamental branes. This dual wrapping rule can be understood by the presence of Kaluza–Klein (KK) monopoles.

We next extend our analysis to the supersymmetric non-standard solitonic branes with less than or equal to two transverse directions whose existence is suggested by supergravity. These brane configurations have been classified in our previous work using the criterion mentioned above [7]. In this Letter we will show that these non-standard solitons are precisely obtained by applying the same dual wrapping rule as in the case of the standard solitons. The fact that this wrapping rule works for the non-standard solitons as well cannot be explained by the presence of the standard KK monopole alone. Instead, it suggests the existence of a class of generalized KK monopoles in ten-dimensional string theory.

2. Doubled geometry

The only fundamental brane in ten dimensions is the fundamental string. It couples to the background metric $g_{\mu\nu}$ via a Nambu–Goto term and to the NS–NS two-form potential $B_{\mu\nu}$ via a WZ term. Schematically, we have

$$\mathcal{L}^{D=10}(\text{Fundamental String}) = T\sqrt{-g} + B_2, \quad (1)$$

where T is the string tension. The first term at the r.h.s. is the Nambu–Goto term containing the determinant of the pull-back of $g_{\mu\nu}$. The second term, where we have used form notation, is the WZ term containing the pull-back of $B_{\mu\nu}$. The special thing about fundamental branes is that their brane tension T is independent of the string coupling constant $g_s = \langle e^\phi \rangle$ with ϕ being the dilaton. In general the tension T of a brane may scale like $T \sim e^{\alpha\phi}$ in terms of an integer number $\alpha \leq 0$. This leads to a classification of branes according to α^3 :

$$\begin{aligned} \alpha = 0: & \text{ Fundamental Branes,} \\ \alpha = -1: & \text{ D-branes,} \\ \alpha = -2: & \text{ Solitonic Branes, } \dots, \text{ etc.} \end{aligned} \quad (2)$$

Another way of classifying branes is according to the number of transverse directions. As already mentioned in the introduction we will call branes with more than two transverse directions “standard” and branes with less than or equal to two transverse directions “non-standard”. Amongst the non-standard branes we will adapt the following nomenclature:

$$\# \text{ transverse directions} = 2: \text{ defect-branes,}$$

Table 1

Upon applying the fundamental wrapping rule (6) one obtains in each dimension a singlet fundamental string and a T-duality vector of fundamental 0-branes.

Fp-brane	IIA/IIB	9	8	7	6	5	4	3
0		2	4	6	8	10	12	14
1	1/1	1	1	1	1	1	1	1

$$\# \text{ transverse directions} = 1: \text{ domain-walls,}$$

$$\# \text{ transverse directions} = 0: \text{ space-filling branes.} \quad (3)$$

Restricting ourselves first to fundamental branes we not only have the fundamental string in $D < 10$ dimensions but also fundamental 0-branes, i.e. wrapped strings. They can be attached to the fundamental string and the corresponding WZ term gets accordingly modified with extra world-volume scalars that satisfy a self-duality condition [9]:

$$\mathcal{L}_{\text{WZ}}^{D \leq 10}(\text{Fundamental String}) = B_2 + \eta^{AB} \mathcal{F}_{1,A} B_{1,B}. \quad (4)$$

Here $B_{1,A}$ are the NS–NS 1-forms and $\mathcal{F}_{1,A} = db_{0,A} + B_{1,A}$ are the 1-form world-volume curvatures of the extra scalars $b_{0,A}$. Both transform as a vector, indicated by the index A , under the T-duality group $\text{SO}(d, d)$ with $d = 10 - D$. The number of extra scalars is twice the number of compactified dimensions in line with doubled geometry [9]. Due to the self-duality condition that these scalars satisfy we obtain $(D - 2) + 1/2 \cdot 2(10 - D) = 8$ worldvolume degrees of freedom, where $D - 2$ is the number of transverse scalars. This is the correct number that fits into a scalar supermultiplet. The WZ term for the fundamental 0-branes themselves does not contain extra scalars and is given by (omitting the explicit vector-index A)

$$\mathcal{L}_{\text{WZ}}^{D \leq 10}(\text{Fundamental 0-Branes}) = B_1. \quad (5)$$

In summary, in D dimensions we have a T-duality vector of fundamental 0-branes and a singlet fundamental string.

Alternatively, the above counting of branes is obtained by applying the following wrapping rule for fundamental branes:

$$\begin{aligned} \text{wrapped} & \rightarrow \text{doubled,} \\ \text{unwrapped} & \rightarrow \text{undoubled.} \end{aligned} \quad (6)$$

The doubling of branes under wrapping is due to the fact that in each dimension there is an extra fundamental 0-brane resulting from the reduction of a pp-wave. This is precisely the manifestation of T-duality. Starting from a single fundamental string in ten dimensions (either IIA or IIB) one obtains the correct number of fundamental branes in each dimension by applying the fundamental wrapping rule (6) for each compactified dimension, see Table 1.

We next consider the D-branes. In $D = 10$ dimensions fundamental strings can end on D-branes and, accordingly, the WZ term gets deformed by an extra Born–Infeld worldvolume vector b_1 , with 2-form curvature $\mathcal{F}_2 = db_1 + B_2$:

$$\mathcal{L}_{\text{WZ}}^{D=10}(\text{D-branes}) = e^{\mathcal{F}_2} C. \quad (7)$$

Here C stands for the formal sum of all RR potentials which are of odd rank for IIA and of even rank for IIB string theory. In [6] we derived the T-duality-covariant expression of the D-brane WZ terms in $D < 10$ dimensions. Since now both wrapped and unwrapped fundamental strings can end on the D-branes we get a further deformation by the extra worldvolume scalars $b_{0,A}$ [6]:

$$\mathcal{L}_{\text{WZ}}^{D \leq 10}(\text{D-branes}) = e^{\mathcal{F}_2} e^{\mathcal{F}_{1,A}} \Gamma^A C, \quad (8)$$

where Γ^A are the gamma-matrices of $\text{SO}(d, d)$. The reason for the existence of the general expression (8) is that in any dimension the

³ We do not consider instantons.

Table 2

Upon applying the D-brane wrapping rule (10) one obtains in each dimension a T-duality spinor of D-branes.

Dp-brane	IIA/IIB	9	8	7	6	5	4	3
0	1/0	1	2	4	8	16	32	64
1	0/1	1	2	4	8	16	32	64
2	1/0	1	2	4	8	16	32	64
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
8	1/0	1						
9	0/1							

fundamental potentials transform as a singlet (2-form) and vector (1-form) under T-duality while the D-brane potentials transform as (chiral) spinor representations of the same duality group. We have omitted these spinor indices in Eq. (8).

At first sight the WZ term (8) does not seem to lead to the correct counting of worldvolume degrees of freedom. For any Dp-brane the Born–Infeld vector corresponds to $p - 1$ degrees of freedom. Considering also the $D - p - 1$ embedding scalars one needs only d extra scalars to fill the bosonic sector of a vector multiplet in $p + 1$ dimensions. Instead, there are $2d$ scalar fields $b_{0,A}$, that is twice too many. Unlike in the case of the fundamental string one can this time not rescue the situation by imposing a self-duality condition on the extra scalars. Luckily, it turns out that the above counting is too naive. The expression (8) stands for the WZ term for a whole spinor representation of D-branes and it is enough to show that a single spinor component representing the WZ term of a particular D-brane contains only half of the $2d$ extra scalars. To show this, it is enough to expand (8) and consider only the first $\mathcal{F}_{1,A}$ term. For a given p we obtain

$$C_{p+1,\alpha} + \mathcal{F}_{1,A} (\Gamma^A)_{\alpha}^{\beta} C_{p,\beta} + \dots, \quad (9)$$

where α is an $SO(d,d)$ spinor index. Using an $SO(d,d)$ light-cone basis, where the light-cone directions are denoted as $A = (1\pm, 2\pm, \dots, d\pm)$, one can show that for a given value of the spinor index α , for any fixed $n = 1, \dots, d$, only one of the two matrices $(\Gamma^{n\pm})_{\alpha}^{\beta}$ gives a non-zero result when acting on a chiral spinor. The detailed proof can be found in [7]. This shows that for any given Dp-brane only half of the $2d$ extra scalars $b_{0,A}$ actually occur, and this results in the correct number of degrees of freedom for a $(p + 1)$ -dimensional worldvolume vector multiplet.

In summary, in each dimension $D < 10$ we have a T-duality spinor of D-branes of dimension 2^{d-1} . These D-branes can be obtained by applying the following D-brane wrapping rule:

$$\begin{aligned} \text{wrapped} &\rightarrow \text{undoubled}, \\ \text{unwrapped} &\rightarrow \text{undoubled}. \end{aligned} \quad (10)$$

Starting from the D-branes of ten-dimensional IIA or IIB string theory and using this wrapping rule one obtains the correct number of D-branes in $D < 10$ dimensions, see Table 2.

Unlike the fundamental wrapping rule the D-brane wrapping rule is self-contained, i.e. it does not need the assistance of gravitational solutions such as the pp-wave. The D-brane sector is also closed under duality in the sense that the dual of a D-brane is again a D-brane. This is not the case for fundamental branes which are dual to solitonic branes. Finally, we note that all potentials that couple to the supersymmetric fundamental branes and D-branes occur in the decomposition

$$\text{U-duality} \supset SO(d,d) \times \mathbb{R}^+ \quad (11)$$

of the U-duality representations according to which the potentials of maximal supergravity transform. The type of brane, i.e. the value of α in the tension $T = (g_s)^{\alpha}$, is determined by the weight of the

Table 3

Upon applying the dual wrapping rule (12) and restricting to solitonic branes with more than two transverse directions one obtains in each dimension a singlet solitonic $(D - 5)$ -brane and a T-duality vector of solitonic $(D - 4)$ -branes. The prime indicates that the corresponding 5-brane has a six-dimensional worldvolume tensor multiplet.

Sp-brane	IIA/IIB	9	8	7	6	5	4	3
0						1	12	
1					1	10		
2				1	8			
3			1	6				
4		1	4					
5	1'/1	1' + 1						

potential under the \mathbb{R}^+ -scaling symmetry. In particular, we find that the U-duality representations of the high-form potentials of rank $D - 2$, $D - 1$ and D under the decomposition (11) give rise to spinor representations of T-duality corresponding to D-branes with less than or equal to two transverse directions.

This concludes our discussion of how fundamental branes and D-branes probe the doubled geometry structure. Requiring that for each brane the corresponding dual brane also belongs to string theory it is natural to include solitons in our discussion since they are dual to the fundamental branes. In the next section we will therefore extend our analysis to string solitons.

3. Solitons and dual doubled geometry

We first restrict ourselves to solitonic branes with more than two transverse directions. Requiring that for every fundamental brane there is a dual solitonic brane one finds that the following dual wrapping rule must be introduced:

$$\begin{aligned} \text{wrapped} &\rightarrow \text{undoubled}, \\ \text{unwrapped} &\rightarrow \text{doubled}. \end{aligned} \quad (12)$$

The doubling of branes when unwrapped is due to the fact that in each dimension there is an extra solitonic $(D - 4)$ -brane resulting from the reduction of a KK monopole. Starting from the standard NS–NS five-brane of ten-dimensional IIA or IIB string theory and using the dual wrapping rule (12) one obtains a singlet solitonic $(D - 5)$ -brane and a T-duality vector of solitonic $(D - 4)$ -branes in each dimension $D < 10$, see Table 3.

Unlike in the case of fundamental branes we find that the class of solitonic branes, like in the case of D-branes, extends to include non-standard branes, i.e. branes with less than or equal to two transverse directions, as well. Under the decomposition (11) of the supergravity fields the string solitons (or S-branes) organize themselves as anti-symmetric tensor representations of the T-duality group, so that in D dimensions an Sp-brane has $(p + d - 5)$ indices, see Table 4 [7]. In each dimension it is understood that the number m of anti-symmetric indices runs from $m = 0$ to $m = d$, which means that the highest possible value of p is 5. Furthermore, the representation with the maximum number of indices – that is d indices – splits into a self-dual and an anti-selfdual representation of the T-duality group. The self-dual representation describes solitonic five-branes with a worldvolume vector multiplet while the anti-selfdual representation involves solitonic five-branes with a worldvolume tensor multiplet.

Unlike the singlet and vector solitons, it turns out that not all components of the higher-rank anti-symmetric tensor representations correspond to supersymmetric solitons. To understand this it is enough to consider only the leading and subleading term in the WZ term of a solitonic Sp-brane which takes the schematic form ($m = p + d - 5$) [7]

$$D_{A_1 \dots A_m} + \overline{\mathcal{G}(C)} \Gamma_{A_1 \dots A_m} C + \dots \quad (13)$$

Table 4

String solitons transform as anti-symmetric tensor representations of the T-duality group. The number m of anti-symmetric indices runs from $m = 0$ to $m = d$.

$S(D-5)$ -brane
$[S(D-4)\text{-brane}]_A$
$[S(D-3)\text{-brane}]_{AB}$
$[S(D-2)\text{-brane}]_{ABC}$
$[S(D-1)\text{-brane}]_{ABCD}$

Here D is the solitonic target space potential, C is the formal sum of RR target space gauge fields and $\mathcal{G}(c)$ is the formal sum of worldvolume RR n -form gauge fields c . Each n -form represents a possible D-brane ending on the soliton. Note that both the target space potentials C and the worldvolume potentials c transform as spinors under T-duality. In general there are too many worldvolume potentials to fit a supermultiplet. We therefore need to limit the number of worldvolume gauge fields as much as possible such that the independent ones do fit into a supermultiplet. One way to restrict this number is by imposing worldvolume duality conditions. It turns out that this is not enough [7]. To obtain a supersymmetric soliton one also needs to restrict the number of T-duality directions in the anti-symmetric tensor representation such that the $\Gamma_{A_1 \dots A_m}$ matrix projects out the correct number of worldvolume gauge fields. Using a lightcone basis $A = (1 \pm, 2 \pm, \dots, d \pm)$ we found that only the components

$$[ABC \dots] = [m \pm n \pm p \pm \dots] \quad \text{with } m \neq n \neq p \neq \dots \quad (14)$$

are supersymmetric. For more details, we refer to [7]. This leads to a precise prediction of the number of supersymmetric solitons in each dimension. For instance, in $D = 6$ dimensions there are solitonic domain-walls transforming in the three-index anti-symmetric tensor representation **56** of the $SO(4, 4)$ T-duality group. This representation occurs in the decomposition (11) of a 5-form gauge potential that transforms in the **144** of the $SO(5, 5)$ U-duality group. According to the counting rule (14) only the directions $1 \pm 2 \pm 3 \pm$, $1 \pm 2 \pm 4 \pm$, $1 \pm 3 \pm 4 \pm$ and $2 \pm 3 \pm 4 \pm$ correspond to supersymmetric solitons. We therefore find that only 32 out of the **56** configurations are supersymmetric.

Remarkably, precisely the same numbers of supersymmetric solitons are obtained by simply extending the dual wrapping rule (12) to the non-standard solitons as well, see Table 5. This includes the 32 supersymmetric solitonic domain-walls in $D = 6$ dimensions mentioned in the example above. This extension is non-trivial in the sense that not only the singlet soliton doubles when unwrapped but the other solitons double as well when unwrapped. Whereas the doubling of the singlet soliton can be understood by the presence of a singlet KK monopole in each dimension, a similar explanation for the doubling of the other solitons is not available. We will discuss a possible interpretation of this result in the next section.

4. Generalized Kaluza–Klein monopoles

To understand the wrapping rule for standard solitons it is enough to consider the standard KK monopole only. In $D = 10$ dimensions the monopole solution is characterized by 6 worldvolume, one isometry and three transverse directions. To obtain a brane one must reduce over the isometry direction which leads to a solitonic S5-brane. The KK monopole is magnetically charged with respect to the KK vector which is represented by off-diagonal components of the metric. Formally, one may therefore say that

Table 5

By applying the dual wrapping rule (12) to both standard and non-standard solitons one obtains precisely the number of supersymmetric solitons predicted by the counting rule (14). The prime indicates 5-branes with a six-dimensional worldvolume tensor multiplet.

Sp-brane	IIA/IIIB	9	8	7	6	5	4	3
0						1	12	84
1					1	10	60	280
2				1	8	40	160	560
3			1	6	24	80	240	
4		1	4	12	32	80		
5	1'/1	1' + 1	2' + 2	4' + 4	8' + 8			

the KK monopole is electrically charged with respect to the dual graviton. Although a dual graviton mixed-symmetry tensor $D_{7,1}$ can only be defined at the linearized level, see e.g. [10], for the present purposes it is convenient to introduce such a potential as an organizing principle. Upon reduction to $D = 9$ dimensions a mixed-symmetry field $D_{7,1}$ gives rise to both a 7-form and a 6-form potential. Only the 6-form potential is dual to the KK vector and corresponds to the KK monopole. The 7-form potential is dual to the KK scalar and does not correspond to a supersymmetric soliton. We therefore need to restrict the possible reductions of $D_{7,1}$ with the condition that when the index after the comma in 7, 1 is internal also one of the indices before the comma has to be internal. The reduction to $D = 9$ dimensions of the NS–NS solitonic 5-brane together with the $D = 10$ KK monopole, represented by the mixed-symmetry tensor $D_{7,1}$, then leads to the desired dual wrapping rule (12) (here \sharp denotes the internal direction)

$$D_6 \rightarrow D_{5\sharp}, D_6,$$

$$D_{7,1} \rightarrow D_{6\sharp,\sharp}. \quad (15)$$

This yields a singlet S4-brane and an $SO(1, 1)$ vector of S5-branes, one with a vector and one with a tensor multiplet. This works for any dimension. For instance, reducing to $D = 7$ dimensions we obtain

$$D_6 \rightarrow D_{3ijk}(1), D_{4ij}(3),$$

$$D_{7,1} \rightarrow D_{4ijk,i}(3), \quad (16)$$

where $i = 1, 2, 3$ is a $GL(3)$ index. The number between brackets indicates the number of potentials. Again we obtain a singlet S2-brane and an $SO(3, 3)$ T-duality vector of S3-branes.

We next extend the analysis to include the non-standard solitons. For the dual wrapping rule to work in this case as well, we need an extra inflow of branes from objects which we call generalized KK monopoles. We represent these extra objects by mixed-symmetry fields, that a priori can be of the generic form $D_{m,n,p,\dots}$ for $m \geq n \geq p$ non-negative integers that denote the number of separately anti-symmetric indices. Surprisingly, the following set of fields suffices:

$$D_{6+n,n}, \quad n = 0, 1, 2, 3, 4, \quad (17)$$

where $6 + n$, n refers to the symmetries corresponding to a Young tableau with two columns, one with $6 + n$ entries and a second one with n entries. This includes the fields D_6 (the dual NS–NS 2-form), $D_{7,1}$ (the dual graviton), $D_{8,2}$ (which is another dual to the NS–NS 2-form) together with the higher-rank fields $D_{9,3}$ and $D_{10,4}$. The rule for all fields is that they give rise to supersymmetric branes only when the n indices on the right of the comma in $D_{6+n,n}$ are compactified along directions on which also n of the $6 + n$ indices on the left of the comma are compactified. This reduction rule implies that there are no solitonic branes with a worldvolume dimension higher than 6.

The restricted reduction rule for the mixed-symmetry fields (17) yields exactly the right number of additional solitons such that the dual wrapping rule (12) works. For instance, the reduction to $D = 6$ dimensions yields the following potentials ($i = 1, 2, 3, 4$ is a $GL(4)$ index):

$$\begin{aligned} D_6 &\rightarrow D_{2ijkl}(1), D_{3ijk}(4), D_{4ij}(6), D_{5i}(4), D_6(1), \\ D_{7,1} &\rightarrow D_{3ijkl,i}(4), D_{4ijk,i}(12), D_{5ij,i}(12), D_{6i,i}(4), \\ D_{8,2} &\rightarrow D_{4ijkl,ij}(6), D_{5ijk,ij}(12), D_{6ij,ij}(6), \\ D_{9,3} &\rightarrow D_{5ijkl,ijk}(4), D_{6ijk,ijk}(4), \\ D_{10,4} &\rightarrow D_{6ijkl,ijkl}(1). \end{aligned} \quad (18)$$

This yields precisely the sequence of 1, 8, 24, 32, 16 potentials that can be found in the $D = 6$ column of Table 5. The other dimensions work in the same way.

It is not clear what the precise status of the mixed-symmetry fields (17) for $n = 2, 3, 4$ is. One point of view is to consider these fields as a formal framework to get a handle on the properties of the non-standard solitons after reduction. According to this point of view they should not themselves be associated with objects in ten-dimensional string theory. A more exciting possibility is that the mixed-symmetry potentials $D_{6+n,n}$ for $n = 2, 3, 4$ can be associated with non-standard KK monopoles in string theory in the same way that the mixed-symmetry tensor $D_{7,1}$ encodes information about the standard KK monopole. It is suggestive to conjecture that the mixed-symmetry potentials $D_{6+n,n}$ represent supersymmetric solutions with 6 worldvolume directions, n isometry directions and $4 - n$ transverse directions. For $n = 0$ this is the NS-NS 5-brane and for $n = 1$ this is the standard KK monopole. The $n = 2, 3, 4$ cases can probably be represented as supersymmetric single brane solutions by uplifting the lower-dimensional solitons. However, since they have less than or equal to 2 transverse directions they will not be well-defined by themselves. For instance, in the case of 2 transverse directions it is likely that one should consider multiple brane configurations to obtain finite energy solutions. This requires a further investigation.

5. Conclusions

In this Letter we first mentioned that the wrapping rule for fundamental branes, to be consistent with T-duality, requires extra 0-branes originating from the reduction of the pp-wave. The D-branes are consistent by themselves and their wrapping rule does not require any additional object. The fundamental branes are mapped under duality to the standard solitons. Accordingly, the wrapping rule for these standard solitons requires the dual of the pp-wave which is the KK-monopole.

We next extended the analysis to the supersymmetric non-standard solitons which have been classified using supergravity input. Remarkably, these non-standard solitons result from the same dual wrapping rule that leads to the standard solitons. This rule can, however, not be explained by another use of the KK monopole. Other objects are needed and we showed that the corresponding fields are a limited number of mixed-symmetry tensors given in (17). Remarkably, these solitonic mixed-symmetry fields are all

contained in the solitonic sector of the spectrum of the very-extended Kac-Moody algebra E_{11} [11]. It will be interesting to see whether this E_{11} algebra can play a guiding role in understanding the organizing principle here.

One may extend the present analysis by considering branes whose tension scales as $(g_s)^\alpha$ with $\alpha \leq -3$. The first objects to consider are the ones with $\alpha = -3$. In ten-dimensional string theory there is only one such object and that is the S-dual of the D7-brane. We will call these exceptional branes E-branes. It turns out that E-branes occur in any dimension as tensor-spinor representations of the T-duality group [12]. The supersymmetric ones can be classified by considering their WZ term. To obtain the lower-dimensional supersymmetric E-branes from ten dimensions one needs the following exotic wrapping rule [12]

$$\begin{aligned} \text{wrapped} &\rightarrow \text{doubled}, \\ \text{unwrapped} &\rightarrow \text{doubled}. \end{aligned} \quad (19)$$

To realize this wrapping rule one needs a further inflow of branes resulting from other objects in string theory. A similar wrapping rule explaining the occurrence of branes in lower dimensions with $\alpha \leq -4$ does not seem to exist [12].

In summary, the classification of supersymmetric branes in lower dimensions with $0 \leq \alpha \leq -3$, suggested by supergravity, can be reproduced by a set of simple wrapping rules. These rules suggest the existence of a set of additional objects in ten-dimensional string theory. It remains to be seen whether these objects indeed have a meaning within string theory.

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